Ch 1 and 3: Factor of Safety

$$FS = \frac{Strength}{Max. Load} = \frac{S_y}{\sigma_{max}}$$

Axial Members: Normal Strain and Normal Stress

$$\varepsilon = \frac{\Delta}{L}; \quad \sigma = \frac{P}{A}; \quad \sigma = E\varepsilon, \text{ for } \sigma < S_y$$

$$\Delta = \frac{PL}{AE}; \quad P = \frac{EA}{L}\Delta; \quad k = \frac{EA}{L}$$

Poisson

$$\nu = -\frac{\varepsilon_T}{\varepsilon} = -\frac{\Delta b/b}{\Delta/L}$$

$$= -\frac{\text{transverse strain normal to applied stress}}{\text{direct strain in direction of applied stress}}$$

Energy Density

$$U_D = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\frac{\sigma^2}{E} = \frac{1}{2}E\varepsilon^2$$

Resilience $U_R = \frac{1}{2}\frac{S_y^2}{E}$

Thin-walled Torsion Member

$$\gamma = \frac{R\theta}{L}; \quad \tau = \frac{T}{2\pi R^2 t}$$

$$\tau = G\gamma, \text{ for } \tau < \tau_y$$

$$\theta = \frac{TL}{2\pi R^3 tG}$$

For a slab under shear force V: $\tau = \frac{V}{A}$

Complementary Shear Stress τ: At a point, the shear stress on perpendicular planes are equal.

Normal/Shear Stiffness Strength relationships

$$G = \frac{E}{2(1+\nu)}; \quad \tau_y = \frac{S_y}{\sqrt{3}}$$

Hooke's Law, in General:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}; \quad \gamma_{zx} = \frac{\tau_{zx}}{G}; \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

Plane Stress: No stress in z-direction (out-of-plane)
Plane Strain: No strain in z-direction (out of plane)

Ch. 4 Axial Members

See table below

Deflection of Point on a simple truss:

- Draw extensions
- Draw perpendiculars from extensions
- Intercept is new location of Point.

Energy Method:

$$U_{total} = \sum U_i = \sum \frac{1}{2}(P\Delta)_i = \sum \frac{1}{2}\left(\frac{P^2L}{AE}\right)_i = \sum \frac{1}{2}\left(\frac{EA}{L}\Delta^2\right)_i$$

Work due to load F, displacing δ: $W = \frac{1}{2}F\delta$

Displacement of a Point: $\delta = \frac{2U}{F}$;

Force to cause displacement: $F = \frac{2U}{\delta}$

Thin-Walled Pressure Vessels

Cylindrical $\sigma_H = \frac{pR}{t}$; $\sigma_L = \frac{pR}{2t}$

$$\varepsilon_H = \frac{\sigma_H}{E} - \nu\frac{\sigma_L}{E} = \frac{pR}{tE}\left(1 - \frac{\nu}{2}\right);$$

$$\varepsilon_L = \frac{\sigma_L}{E} - \nu\frac{\sigma_H}{E} = \frac{pR}{tE}\left(\frac{1}{2} - \nu\right)$$

$$\varepsilon_t = -\nu\frac{\sigma_H}{E} - \nu\frac{\sigma_L}{E} = -\nu\frac{3pR}{2tE}$$

Spherical $\sigma_S = \frac{pR}{2t}$

$$\varepsilon_S = \frac{\sigma_S}{E} - \nu\frac{\sigma_s}{E} = \frac{pR}{2tE}(1-\nu);$$

$$\varepsilon_t = -\nu\frac{\sigma_S}{E} - \nu\frac{\sigma_s}{E} = -\nu\frac{pR}{tE}$$

Ch. 5 Torsion Members

$$\tau(r) = \frac{Tr}{J}; \quad \tau_{max} = \frac{TR}{J}; \quad \theta = \frac{TL}{JG}$$

$$J = \int_A r^2 dA; \qquad J_{solid} = \frac{\pi R^4}{2} = \frac{\pi D^4}{32};$$

$$J_{thick} = \frac{\pi}{2}\left(R_o^4 - R_i^4\right); \quad J_{thin} = 2\pi R^3 t$$

Power

$$P = T\omega \quad [\text{W}] = [\text{N}\cdot\text{m}][\text{rad/s}]$$

$$T[\text{lb}-\text{ft}] = \frac{33000 \times hp}{2\pi N}; \quad N\ [\text{rpm}]$$

$$T[\text{lb}-\text{in.}] = \frac{63000 \times hp}{N}; \quad N\ [\text{rpm}]$$

Constant P,A,E	$\sigma = \frac{P}{A}$	$\varepsilon = \frac{\Delta}{L} = \frac{\sigma}{E}$	$\Delta = \varepsilon L = \frac{PL}{AE}$
Discretely Varying	$\sigma_i = \frac{P_i}{A_i}$	$\varepsilon_i = \frac{\Delta_i}{L_i} = \frac{\sigma_i}{E_i}$	$\Delta = \sum\left(\frac{PL}{AE}\right)_i$
Cont. Varying	$\sigma(x) = \frac{P(x)}{A(x)}$	$\varepsilon(x) = \frac{du}{dx} = \frac{\sigma(x)}{E(x)}$	$\Delta = \int_L \frac{P(x)}{A(x)E(x)}dx$

For all problems:

- **Equilibrium**
- **Compatibility**

Ch 6: Beams

Bending Stress (bending about *z*-axis):

$$\sigma(y) = -\frac{M_z y}{I_z}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{My_{max}}{I} = \frac{M}{Z}$$

Moment of Inertia of common shapes

Solid Rect.	$I = \frac{bd^3}{12}$
Solid Circle	$I = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$

Parallel Axis Theorem

$I = I_o + Ad^2$; d measured from (bending) axis

Section Modulus about z-axis:

$$Z_z = \frac{I_z}{c} = \frac{I_z}{y_{max}}$$

I-Beams W: wide flange; S: standard

W # x ##: Nominal Depth x lb (per foot)

Beam Deflection

$$v''(x) = \frac{M(x)}{EI}$$

$$v'(x) = \frac{1}{EI}\left[\int M(x)dx + c_1\right] = \theta(x)$$

$$v(x) = \frac{1}{EI}\left[\int \left(\int M(x)dx\right) dx + c_1 x + c_2\right]$$

$v > 0$: up; $v < 0$: down; $v' > 0$ positive slope

Solve c_1 and c_2 by applying *Boundary Conditions*:

Pin/Roller	$v = 0$
Wall	$v = 0\,, \quad v' = 0$
Enforced displacement at x_o	$v(x_o)$ = value

Shear Stress (shear force in *y*-direction):

$$\tau(y) = \frac{VA^* y^*}{It}$$

Maximum τ for Common Shapes.

Solid Rect.	$\tau_{max} = \frac{3}{2}\frac{V}{A}$
Solid Circle	$\tau_{max} = \frac{4}{3}\frac{V}{A}$
Hollow Circle	$\tau_{max} = \frac{4}{3}\frac{V}{A}\left[\frac{R_o^2 + R_o R_i + R_i^2}{R_o^2 + R_i^2}\right]$

Shear Flow

$$q = \tau t = \frac{VA^* y^*}{I}$$

Shear Force parallel to neutral axis given *y*-value over beam length Δs:

$$F_s = \tau(t\Delta s) = q\Delta s$$

Shear Force per unit length of beam parallel to neutral axis given *y*-value:

$$\frac{F_s}{\Delta s} = \tau t = q$$

Beam Design:

$$\sigma_{max} < \sigma_{allowable}$$

$$\tau_{max} < \tau_{allowable}$$

$$\delta_{max} = |v_{max}| < \tau_{allowable}$$

Ch 7: Combined Loading

- σ is caused by Axial Loading (struts, columns), Bending Moments (beams) and Pressure in Pressure Vessels
- τ is caused by Torsion (shafts) and Shear Stress (beams)

1. Determine Internal Loads that act on a cross-section.
2. Determine Stresses caused by Loads
3. Determine where those stresses act.
4. Draw the stress element … the material point (as a cube/square) and the stresses that act on it.

Ch 8. Stress Transformation

Given initial stress state: σ_x, σ_y, τ_{xy}

Rotation by θ CCW:

$$\sigma_{x'}(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'}(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'}(\theta) = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stresses:

$$\sigma_I, \sigma_{II} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Principal Directions:

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$; match θ_I to σ_I; θ_{II} to σ_{II}

Maximum Shear Stress (in *x*-*y* plane)

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Maximum Shear Stress on a face defined by:

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Mohr's Circle

- Plot $X(\sigma_x, -\tau_{xy})$, $Y(\sigma_y, +\tau_{xy})$
- Circle centered at (σ_{ave}, 0), radius = τ_{max})
- Ends of diameter represent *x*-*y* faces of element.
- Rotate θ in element, Rotate 2θ in circle

Ch 9. Failure Theories

Maximum Normal Stress Criterion; Brittle Materials

Tensile: $\sigma_I \le S_u$

Compressive: $|\sigma_{II}| \le S_c$

Maximum Shear Stress Criterion (Tresca); **Ductile Materials**

$$\tau_{max} \le \tau_y$$

$$\tau_{max} = \max\left[\frac{|\sigma_I - \sigma_{II}|}{2}, \frac{|\sigma_I|}{2}, \frac{|\sigma_{II}|}{2}\right]$$

Shear Yield Strength:

$$\tau_y = \frac{S_y}{2}$$

Von Mises Failure Criterion (Max. Distortion Energy); **Ductile Materials**

$$\sigma_o \le S_y$$

$$\sigma_o = \left[\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}{2}\right]^{1/2}$$

Plane Stress

$$\sigma_o = [\sigma_I{}^2 - \sigma_I\sigma_{II} + \sigma_{II}{}^2]^{1/2}$$

$$\sigma_o = \left[\sigma_x{}^2 - \sigma_x\sigma_y + \sigma_y{}^2 + 3\tau_{xy}{}^2\right]^{1/2}$$

Shear Yield Strength:

$$\tau_y = \frac{S_y}{\sqrt{3}}$$

Ch 10. Buckling

Critical Buckling Load:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

Buckling Strength:

$$\sigma_{cr} = \frac{\pi^2 EI}{AL_e^2}$$

Buckling can occur about either the *y*- or *z*-axis (assuming *x*-axis is along axial direction).

For buckling *about the z-axis*:

$$P_{cr,z} = \frac{\pi^2 EI_z}{L_{e,z}^2}$$

I_z is *I*-about *z*-axis, $L_{e,z}$ is effective length for *z*-axis buckling.

For buckling *about the y-axis*:

$$P_{cr,y} = \frac{\pi^2 EI_y}{L_{e,y}^2}$$

Effective Length, L_e

Pinned-Pinned	$L_e = L$
Fixed-Fixed	$L_e = 0.5L$
Free-Fixed	$L_e = 2L$
Pinned-Fixed	$L_e = 0.7L$

Radius of Gyration

$$r = \sqrt{\frac{I}{A}}\ ; \quad r_z = \sqrt{\frac{I_z}{A}}\ ; \quad r_y = \sqrt{\frac{I_y}{A}}$$

So that:

$$\sigma_{cr} = \frac{\pi^2 E r^2}{L_e^2}$$