Engr 154 Exam Note Sheet updated s20

2/2 Rectilinear Motion

Fundamental Kinematics Equations:

(1) $v = \frac{ds}{dt} = \dot{s}$

(2) $a = \frac{dv}{dt} = \dot{v} = \frac{d^2s}{dt^2} = \ddot{s}$

(3) $v\,dv = a\,ds$

When a is constant:

$v = v_o + at$

$s = s_o + v_o t + \frac{1}{2}at^2$

$v^2 = v_o^2 + 2a(s - s_o)$

If a is not constant, use one of the fundamental kinematic equations and appropriately integrate/differentiate.

2/5 Normal and Tangential Coordinates (n-t)

$\underline{\mathbf{v}} = v\underline{\mathbf{e_t}} = \rho\dot{\beta}\underline{\mathbf{e_t}}$

$\underline{\mathbf{a}} = \underline{\dot{\mathbf{v}}} = a_t\underline{\mathbf{e_t}} + a_n\underline{\mathbf{e_n}} = \dot{v}\underline{\mathbf{e_t}} + \left\langle \frac{v^2}{\rho} \middle| \rho\dot{\beta}^2 \middle| v\dot{\beta} \right\rangle \underline{\mathbf{e_n}}$

Circular motion:

$\rho \to r, \qquad \dot{\beta} \to \dot{\theta}$;

$v = r\dot{\theta}$

$a_t = \dot{v} = r\ddot{\theta}, \qquad a_n = v^2/r = r\dot{\theta}^2 = v\dot{\theta}$

2/3 Plane Curvilinear Motion

$\underline{\mathbf{v}} = \frac{d\underline{\mathbf{r}}}{dt} = \underline{\dot{\mathbf{r}}}$

$\underline{\mathbf{a}} = \frac{d\underline{\mathbf{v}}}{dt} = \underline{\dot{\mathbf{v}}}$

2/6 Polar Coordinates (r-θ)

$\underline{\mathbf{r}} = r\underline{\mathbf{e_r}}$

$\underline{\mathbf{v}} = v_r\underline{\mathbf{e_r}} + v_\theta\underline{\mathbf{e_\theta}} = \dot{r}\underline{\mathbf{e_r}} + r\dot{\theta}\underline{\mathbf{e_\theta}}$

$\underline{\mathbf{a}} = a_r\underline{\mathbf{e_r}} + a_\theta\underline{\mathbf{e_\theta}} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{\mathbf{e_r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{\mathbf{e_\theta}}$

2/4 Rectilinear Coordinates

$\underline{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$\underline{\mathbf{v}} = \underline{\dot{\mathbf{r}}} = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

$\underline{\mathbf{a}} = \underline{\dot{\mathbf{v}}} = \underline{\ddot{\mathbf{r}}} = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$

Speed (magnitude of velocity): $v = \sqrt{v_x^2 + v_y^2}$

Direction of motion: $\tan\theta = \frac{v_y}{v_x}$

Acceleration $a = \sqrt{a_x^2 + a_y^2}$

2/8 Relative Motion

$\underline{\mathbf{r_A}} = \underline{\mathbf{r_B}} + \underline{\mathbf{r_{A/B}}}$

$\underline{\mathbf{v_A}} = \underline{\mathbf{v_B}} + \underline{\mathbf{v_{A/B}}}$

$\underline{\mathbf{a_A}} = \underline{\mathbf{a_B}} + \underline{\mathbf{a_{A/B}}}$

Projectile Motion

initial position (x_o, y_o), initial velocity v_o:

$a_x = 0 \qquad a_y = -g$

$v_x = v_{o,x} \qquad v_y = v_{o,y} - gt$

$x = x_o + v_{o,x}t \qquad y = y_o + v_{o,y}t - \frac{1}{2}gt^2$

Also: $\left(v_y\right)^2 = \left(v_{o,y}\right)^2 - 2g(y - y_o)$

2/9 Constrained Motion

Cable/Pulley Systems: write cable length(s) – or other dimensions that remain constant – in terms of variable distances. Cable length L is constant; its derivative is:

$\dot{L} = 0 = derivatives\ of\ variable\ distances.$

Alternate: Use differential movement *ds* of a point on cable, and how *ds* promulgated to various points on the cable/around pulleys, etc. $v = ds/dt$.

The same vector can be represented in different coordinate systems (*x*-*y*), (*n*-*t*), (*r*-*θ*).

Lay out the same vector in different coordinate systems. Use vector geometry/trigonometry to determine sides of the vector triangle in different coordinate systems.

$\underline{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = r\underline{\mathbf{e_r}}$

$\underline{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = v\underline{\mathbf{e_t}} = v_r\underline{\mathbf{e_r}} + v_\theta\underline{\mathbf{e_\theta}}$

$\underline{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = a_t\underline{\mathbf{e_t}} + a_n\underline{\mathbf{e_n}} = a_r\underline{\mathbf{e_r}} + a_\theta\underline{\mathbf{e_\theta}}$

The magnitude of a vector is the same in any coordinate system, e.g., the magnitude of the velocity and acceleration are, respectively:

$v = \sqrt{v_x^2 + v_y^2} = v_t = \sqrt{v_r^2 + v_\theta^2}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_t^2 + a_n^2} = \sqrt{a_r^2 + a_\theta^2}$

Chapter 3: Kinetics of Particles

3/4 Rectilinear Motion

$$\sum \underline{\mathbf{F}} = m\underline{\mathbf{a}}$$

$$\Sigma F_x = ma_x \quad \Sigma F_y = ma_y \quad \Sigma F_z = ma_z$$

$$|\Sigma \underline{\mathbf{F}}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

3/5 Plane Curvilinear Motion

Rectangular

$$\Sigma F_x = ma_x \qquad a_x = \ddot{x}$$

$$\Sigma F_y = ma_y \qquad a_y = \ddot{y}$$

Normal-Tangential

$$\Sigma F_n = ma_n \qquad a_n = v^2/\rho = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$\Sigma F_t = ma_t \qquad a_t = \dot{v}$$

Polar

$$\Sigma F_r = ma_r \qquad a_r = \ddot{r} - r\dot{\theta}^2$$

$$\Sigma F_\theta = ma_\theta \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

3/6 Work and Kinetic Energy

Work done by force $\underline{\mathbf{F}}$ during displacement $d\underline{\mathbf{r}}$:

$$dU = \underline{\mathbf{F}} \cdot d\underline{\mathbf{r}}$$

Work done by force $\underline{\mathbf{F}}$ during finite motion (1) to (2):

$$U_{1-2} = \int_1^2 \underline{\mathbf{F}} \cdot d\underline{\mathbf{r}} = \int_1^2 (F_x dx) + (F_y dy) + (F_z dz)$$

Work done by constant force $\underline{\mathbf{P}}$ moving distance Δs

$$U_P = \int_1^2 \underline{\mathbf{F}} \cdot d\underline{\mathbf{r}} = P\Delta s$$

Work done <u>by spring force</u> on object

$$U_s = \int_1^2 \underline{\mathbf{F}} \cdot d\underline{\mathbf{r}} = -\frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}k(x_1^2 - x_2^2)$$

Work done by gravity (near earth)

$$U_g = \int_1^2 \underline{\mathbf{F}} \cdot d\underline{\mathbf{r}} = -mg(y_2 - y_1) = mg(y_1 - y_2)$$

Work done by gravity (far from earth)

$$U_g = \int_1^2 \underline{\mathbf{F}} \cdot d\underline{\mathbf{r}} = Gm_e mR^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Work-Kinetic Energy Theorem

$$U_{1-2} = \int_1^2 \underline{\mathbf{F}} \cdot d\underline{\mathbf{r}} = \int_1^2 (F_t ds) = \int_1^2 (ma_t ds) = \int_1^2 (mvdv) = \tfrac{1}{2}m(v_2^2 - v_1^2) = T_2 - T_1$$

Work done by all forces on mass equals ΔT:

$$U_{1-2} = \Delta T$$

Power

$$P = \underline{\mathbf{F}} \cdot \underline{\mathbf{v}} = \frac{dU}{dt}$$

Efficiency:

$e = \frac{P_{out}}{P_{in}}$ Overall Efficiency: $e = e_m e_e e_t$

3/7 Potential Energy

$$V_g = mgh \qquad \Delta V_g = mg(\Delta h)$$

$V_e = \frac{1}{2}kx^2 \qquad \Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2)$; *x=displ. of spring from equilib.*

Work-Energy Equation

Work of all non-conserv. forces U_{nc} equals change in mechanical energy $E = T + V_g + V_e$

$$U_{nc} = \Delta T + \Delta V_g + \Delta V_e = \Delta E$$

$$E_2 = E_1 + U_{nc}$$

3/8 Linear Impulse and Linear Momentum

$$\underline{\mathbf{G}} = m\underline{\mathbf{v}}$$

$\Sigma\underline{\mathbf{F}} = \dot{\underline{\mathbf{G}}} = m\dot{\underline{\mathbf{v}}}$; (*external* forces)

$$\Sigma F_x = \dot{G}_x = m\dot{v}_x; \quad \Sigma F_y = \dot{G}_y; \quad \Sigma F_z = \dot{G}_z$$

$$\int_{t_1}^{t_2} \Sigma\underline{\mathbf{F}}\, dt = \Delta\underline{\mathbf{G}} = m\Delta\underline{\mathbf{v}}$$

$$\underline{\mathbf{G}}_2 = \underline{\mathbf{G}}_1 + \int_{t_1}^{t_2} \Sigma\underline{\mathbf{F}}\, dt$$

Collisions: $\underline{\mathbf{G}}$ *of system* conserved if no external force.

3/9 Angular Impulse and Angular Momentum

$$\underline{\mathbf{H}}_o = \underline{\mathbf{r}} \times m\underline{\mathbf{v}}$$

$$\underline{\mathbf{H}}_o = m(yv_z - zv_y)\hat{\mathbf{i}} + m(zv_x - xv_z)\hat{\mathbf{j}} + m(xv_y - yv_x)\hat{\mathbf{k}}$$

$\Sigma\underline{\mathbf{M}}_O = \dot{\underline{\mathbf{H}}}_o = \underline{\mathbf{r}} \times m\dot{\underline{\mathbf{v}}}$ *(external moments)*

$$\Sigma M_{Ox} = \dot{H}_{Ox}; \quad \Sigma M_{Oy} = \dot{H}_{Oy}; \quad \Sigma M_{Oz} = \dot{H}_{Oz}$$

$$\int_{t_1}^{t_2} \Sigma\underline{\mathbf{M}}_O\, dt = \Delta\underline{\mathbf{H}}_o$$

$$\left(\underline{\mathbf{H}}_o\right)_2 = \left(\underline{\mathbf{H}}_o\right)_1 + \int_{t_1}^{t_2} \Sigma\underline{\mathbf{M}}_O\, dt$$

Collisions: $\underline{\mathbf{H}}_o$ *of system* conserved if no external moment.

3/12 Impact

Coefficient of Restitution

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

Only affects velocity <u>normal</u> to impact plane.

Collisions:

$\underline{\mathbf{G}}$ *of system* conserved if no net external force on system

$\underline{\mathbf{H}}_o$ *of system* conserved if no net external moment

<u>Elastic</u>: objects bounce; energy conserved: $T_i = T_f$

<u>Inelastic</u>: objects stick; energy not cons'd: $\underline{\mathbf{v}}_1' = \underline{\mathbf{v}}_2'$

<u>Impact</u>: objects bounce; energy not cons'd: $e = \frac{v_2' - v_1'}{v_1 - v_2}$

Units

Work: 1 J = 1 N·m

Energy: 1 W= 1 J/s; 1 hp = 550 ft-lb/sec = 746 W

Chapter 5: Kinematics of Planar Motion

5/2 Rotation

Fundamental Kinematics Equations – Angular Motion (analogous to linear motion)

(1) $\omega = \frac{d\theta}{dt} = \dot{\theta}$

(2) $\alpha = \frac{d\omega}{dt} = \dot{\omega} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$

(3) $\omega d\omega = \alpha\, d\theta$

When α is <u>constant</u>:

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

If α is not constant, use one of the fundamental kinematic equations and appropriately integrate/differentiate.

Vector Algebra for motion of Point A about a fixed axis through Point O (relative to Point O)

$\underline{\mathbf{v}_A} = \underline{\boldsymbol{\omega}} \times \underline{\mathbf{r}} = \omega r \underline{\mathbf{e}_t}$ $\quad \underline{\mathbf{r}}$ is ***from*** Pt 0 ***to*** Pt A.

$$\underline{\mathbf{a}_A} = \underline{\mathbf{a}_t} + \underline{\mathbf{a}_n} = \alpha r \underline{\mathbf{e}_t} + \omega^2 r \underline{\mathbf{e}_n}$$

$$= \underline{\boldsymbol{\alpha}} \times \underline{\mathbf{r}} + \underline{\boldsymbol{\omega}} \times (\underline{\boldsymbol{\omega}} \times \underline{\mathbf{r}})$$

5/3 Absolution Motion

- Define geometry in terms of linear & angular positions.
- Take derivative to relate linear & angular velocities.
- Take derivative to relate linear & angular accelerations.

e.g., in a triangle,

$$y = x\tan\theta \ldots \dot{y} = \dot{x}\tan\theta + x\sec^2\theta(\dot{\theta})$$

5/4 Relative Velocity, 5/6 Relative Acceleration

$$\underline{\mathbf{v}_A} = \underline{\mathbf{v}_B} + \underline{\mathbf{v}_{A/B}} = \underline{\mathbf{v}_B} + \underline{\boldsymbol{\omega}_{AB}} \times \underline{\mathbf{r}_{A/B}}$$

$$\underline{\mathbf{a}_A} = \underline{\mathbf{a}_B} + \underline{\mathbf{a}_{A/B}} = \underline{\mathbf{a}_B} + \left(\underline{\mathbf{a}_{A/B}}\right)_t + \left(\underline{\mathbf{a}_{A/B}}\right)_n$$

$$= \underline{\mathbf{a}_B} + \alpha r \underline{\mathbf{e}_t} + \omega^2 r \underline{\mathbf{e}_n}$$

Generally, the vectors are broken up into x- and y-components. Be careful about directions!

5/5 Instantaneous Center of Zero Velocity

- Construct lines perpendicular to two (or more) velocities on a rigid body
- Intersection of constructed lines is ICOZV (Pt C)
- Velocity of any Pt A on the rigid body is equal in magnitude to : $v_A = \omega r_{A/O}$ and is perpendicular to the vector from Pt. O to Pt. A; i.e.,: $\underline{\mathbf{v}_A} = \underline{\boldsymbol{\omega}} \times \underline{\mathbf{r}_{A/O}}$

Chapter 6: Kinematics of Planar Motion (x-y plane)

6/2 General Plane Motion Equations

$$\sum \underline{\mathbf{F}} = m\underline{\mathbf{a}}$$

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

$$|\Sigma \underline{\mathbf{F}}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$a = \sqrt{a_x^2 + a_y^2}$$

Assuming rotation only about z-axis, only need use scalar version of moment, etc.

Moments about Center of Mass Pt. G:

$$\sum M_G = I_G \alpha$$

Moments about <u>non-accelerating</u> or <u>fixed</u> Point O:

$$\sum M_O = I_O \alpha$$

Moments about arbitrary Point P:

$$\sum M_P = I_G \alpha + m a_G d$$

where a_G is the magnitude of acceleration of C.O.M., and d is the perpendicular distance from point P to the line of action of $\underline{a_G}$

Moments about arbitrary Point P:

$$\sum \underline{\mathbf{M}_\mathbf{P}} = I_P \underline{\boldsymbol{\alpha}} + \underline{\boldsymbol{\rho}_G} \times m\underline{\mathbf{a}_\mathbf{p}}$$

where:

$\underline{\boldsymbol{\alpha}}$ is the vector angular acceleration,

$\underline{\boldsymbol{\rho}_G}$ is vector position ***from*** Pt. P to Pt. G

$\underline{\mathbf{a}_\mathbf{p}}$ is vector acceleration of Pt. P

6/3 Translation

$$\sum \underline{\mathbf{F}} = m\underline{\mathbf{a}} \quad ; \quad \sum M_G = 0$$

6/4 Fixed-Axis Rotation

$$\sum \underline{\mathbf{F}} = m\underline{\mathbf{a}} \quad ; \quad \sum M_G = I_G \alpha \quad ; \quad \sum M_O = I_O \alpha$$

6/5 General Motion

$\sum \underline{\mathbf{F}} = m\underline{\mathbf{a}} \quad ; \quad \sum M_G = I_G \alpha \quad ; \quad \sum M_p$ equations

Moments of Inertia for basic shapes:

<u>Slender Bar about COM</u>; $\frac{ml^2}{12}$	<u>Slender Bar about end</u> $\frac{ml^2}{3}$
<u>Cylindrical Disk</u> $\frac{mR^2}{2}$	<u>Sphere</u> $\frac{2}{5}mR^2$

Parallel Axis Theorem: $I = I_G + md^2$

Radius of Gyration: $I = mk^2$