Brief Notes on Significant Figures (Digits)

Compiled/edited by: D.J. Dal Bello, revG, 7/2020

When a measured or calculated quantity is written out, the number of ***Significant Figures*** (***Digits***) indicates how ***precise*** the measurement is, i.e., how well the measurement can be described. When you write a measurement with four significant digits, such as 23.45, you are indicating that you know the first three digits very well, but there is some uncertainty is the last digit. At worst, for 23.45, the uncertainty is ±0.01 (about ±0.04%), which might be considered very ***precise***. ***Precision*** is not ***accuracy***, although they are often confused. Writing the acceleration of gravity on the earth's surface as g = 4.236135862 m/s^2 (10 sig. figs) is very *precise* (±0.000000001), but not *accurate* (close to the accepted value) of 9.81 m/s^2 (3 sig. figs).

In a textbook, problems are generally assumed to be exact (and often given to 3 figures), and answers are generally given to 3 SF (to 4 if the number starts with a "1"). Considering only integers, the smallest 3-digit number is 200. The last digit is uncertain, so the largest error is ±1; the precision is ±0.5%. Considering the uncertainty of "known" measured values in engineering, 0.5% error is very precise. In lab, where *measurements* are actually taken, data should be read to as many digits as the instrument allows. Lab calculations **must** use the rules of significant figures to give appropriate numerical answers.

Understanding and Using Significant Figures [Digits]

From: http://www.phys.unt.edu/PIC/significant_figures.htm Accessed June 26, 2007

Editorial comments by D.J. Dal Bello in square brackets [DJD]. Reformatted.

In scientific work, most numbers are measured quantities and thus are not exact. All measured (continuous) quantities are limited in significant figures (SF) by the precision of the instrument used to make the measurement. The measurement must be recorded in such a way as to show the *degree of precision* to which it was made – no more, no less. Calculations based on the measured quantities can have no more (or no less) precision than the measurements themselves. The answers to the calculations must be recorded to the proper number of significant figures. To do otherwise is misleading and improper.

Determining Which Figures are Significant [in a given value]

- ***Non-zero digits*** are always significant.
  example: 23.4 g, 234 g and 0.0234, all have 3 SF
- ***Captive zeroes***, those bounded on both sides by non-zero integers, are always significant.
  example: 20.05 has 4 SF; 407 has 3 SF
- ***Leading zeroes***, those not bounded on the left by non-zero integers, are never significant [i.e., numbers less than 1]. Such zeros just set the decimal point; they always disappear if the number is converted to powers-of-10 [scientific] notation.
  example: 0.04 g has 1 SF; 0.00035 has 2 SF. They can be written as $4x10^{-2}$ and $3.5x10^{-4}$, respectively.
- ***Trailing zeroes***, those bounded only on the left by non-zero integers **may or may not** be significant.
  example: 45.0 L has 3 SF; 450 L has only 2 SF [but perhaps 3]; 450. L has 3 SF. [DJD: the decimal point at the end of "450." explicitly indicates its 0 is significant]

  Note: To clarify whether a trailing zero is significant, it is preferable to use scientific notation [or engineering notation] to express the final answer.
  example: 450. L can be expressed as 4.50×10^2 L… whereas 450 L would be expressed as 4.5×10^2 L [in engineering notation: 0.450×10^3 and 0.45×10^3 L].
- ***Exact numbers*** are those not obtained by measurement but ***by definition*** or **by *counting numbers*** of objects. They are assumed to have an unlimited number of significant figs.

Multiplication and Division Involving Significant Figures

Calculations involving only multiplication and/or division of **measured** quantities shall have the same number of significant figures as the fewest possessed by any measured quantity in the calculation.

example: 14.0×3 = 40, not 42, because one of the [measured] multipliers has only one SF.

example: 14.0×3.0 = 42, because one of the [measured] multipliers has only two SF.

example: 14.0/3 = 5, not 4.6, because the [measured] denominator has only one SF.

Addition and Subtraction Involving Significant Figures

Calculations where measured quantities are added or subtracted shall correspond to the position of the last significant figure in any of the measured quantities. That is, the final answer is only as precise as the decimal position of the least precise value [in terms of absolute precision… the decimal place]. The number of significant figures can change during these calculations.

example: 14.16 + 3.2 = 17.36 (this is not the final answer!) 17.4 is the correct final answer

[*example*: 46 + 5.723 = 51.723 (this is not the final answer!) 52 is the correct final answer].

Tips for Rounding Off Numbers

A number is rounded off to the desired number of significant figures by dropping one or more digits to the right. If you are rounding to the tens, hundreds place or higher, you must put zeroes in the lesser places (the ones place, for example) to indicate to what place you have rounded. The following guidelines should be observed when rounding off numbers.

- When the first digit dropped is less than 5, the last digit remains unchanged.
- When the first digit dropped is more than or equal to five, the last digit retained is increased by 1.

example: 243 → 240; 17.9 → 18; 2.25 → 2.3; 2.35 → 2.4

[DJD: you may have learned that if a number ends in exactly 5, and you are rounding, you round to the nearest even number, e.g.: 2.25 → 2.2; 2.35 → 2.4. This rule seems to have been designed for a large quantity of data…where rounding some values up, and others down, will tend to cancel each other out. A single measurement ending in exactly 5 is typically rounded up.]

More Notes

http://www.physics.uoguelph.ca/tutorial/sig_fig/SIG_dig.htm Accessed June 26, 2007

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Keep at Least One Extra Digit in Intermediate Answers

When doing multi-step calculations, *keep at least one more significant digit in intermediate results* than needed in your final answer.

For instance, if a final answer requires two significant digits, then carry at least three significant digits in calculations. If you round-off all your intermediate answers to only two digits, you are discarding the information contained in the third digit, and as a result the *second* digit in your final answer might be incorrect. (This phenomenon is known as “round-off error.”)

The Two Greatest Sins Regarding Significant Digits

1. Writing more digits in a final answer than justified by the number of digits in the data.
2. Rounding-off [**too early**. For example, rounding] to two digits in an intermediate answer, and then writing three digits in the final answer [what I call, “unrounding”. See Sample Problem 1/1 in Meriam and Kraige’s *Statics* text, as well as in their *Dynamics* – keep digits in intermediate answers].

Angles

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There are no set standard rules with regard to angles. Since angles 1.1° and 36,001.1° describe exactly the <u>same</u> direction, traditional significant figures rules do not work. Angles represent how far you have gone around the circle; they represent direction. I recommend the following:

- All final results should be *consistently* written to the same decimal place. Use *at least* one decimal place after the decimal point: write 41.3° and 241.3° (do not apply “3 sig figs”: 41.3° and 241°).
- If final angles are to be written to one place after the decimal point, angles used in *intermediate* calculations should be to *at least* two decimal places (e.g., 41.34°). The value of trigonometric functions can be sensitive to small changes in angle, so even more decimal places would be better.