Ch. 2 Force Systems

Rectangular Components

Vector	$\underline{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath}$
Vector components	$\underline{\mathbf{F_x}} = F_x\hat{\imath}$; $\underline{\mathbf{F_y}} = F_y\hat{\jmath}$
Scalar components (POS or NEG depending on direction wrt +axes)	$F_x = F\cos\theta_x$ $F_y = F\sin\theta_x = F\cos\theta_y$
Magnitude and Direction wrt $+x$	$F = \sqrt{{F_x}^2 + {F_y}^2}$; $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$
Projection of force onto any direction (defined by **n**. Component of **F** in **n**–direction.	**n** = unit vector in direction of interest θ_n = angle between **F** and **n**. $F_n = \underline{\mathbf{F}}\cdot\underline{\mathbf{n}} = F\cos\theta_n = F_x n_x + F_y n_y$
Resultant force	$\underline{\mathbf{R}} = \sum\underline{\mathbf{F}} = (\sum F_x)\hat{\imath} + (\sum F_y)\hat{\jmath}$ $\underline{\mathbf{R}} = R_x\hat{\imath} + R_y\hat{\jmath}$

Moment about a Point O

Position vector: Pt O to LOA of **F**	$\underline{\mathbf{r}} = r_x\hat{\imath} + r_y\hat{\jmath}$
Vector	$\underline{\mathbf{M_O}} = \underline{\mathbf{r}} \times \underline{\mathbf{F}}$
Scalar (+ CCW; – CW)	$M_O = rF\sin\alpha = (r_xF_y - r_yF_x)$ α = angle **from r to F**
Scalar Magnitude	$M_O = Fd$ d = perpendicular distance
Varignon's Thm: If all forces concurrent at a point (e.g., Pt.O)	$\underline{\mathbf{M_O}} = \sum(\underline{\mathbf{r}} \times \underline{\mathbf{F}}) = \underline{\mathbf{r}} \times \sum\underline{\mathbf{F}} = \underline{\mathbf{r}} \times \underline{\mathbf{R}}$
Couple: two equal and opp. forces	Magnitude: $M = Fd$ (CW/CCW) Vector: $\underline{\mathbf{M}} = \underline{\mathbf{r}} \times \underline{\mathbf{F}}$

Resultants (Force-Couple System ... $\underline{\mathbf{R}} - \underline{\mathbf{M_O}}$)

Resultant force	$\underline{\mathbf{R}} = \sum\underline{\mathbf{F}} = (\sum F_x)\hat{\imath} + (\sum F_y)\hat{\jmath}$ $\underline{\mathbf{R}} = R_x\hat{\imath} + R_y\hat{\jmath}$
	$R_x = \sum F_x$; $R_y = \sum F_y$
	$R = \sqrt{{R_x}^2 + {R_y}^2}$
	$\theta_R = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$
Net moment	$\underline{\mathbf{M_O}} = \sum\underline{\mathbf{M_O}} = \sum(\underline{\mathbf{r}} \times \underline{\mathbf{F}}) + \sum\underline{\mathbf{M_{Couple}}}$ $M_O = \sum Fd$
Moving force away from Pt. O	$\underline{\mathbf{R}} = \sum\underline{\mathbf{F}}$ $M_O = \sum Fd = DR = xR_y - yR_x$ D = perpendicular distance to **R** $x\hat{\imath} + y\hat{\jmath}$…from Pt O to LOA of **R**

3D Force Systems

Vector	$\underline{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$ $= F[(\cos\theta_x)\hat{\imath} + (\cos\theta_y)\hat{\jmath} + (\cos\theta_z)\hat{k}]$
Scalar Components	$F_x = F\cos\theta_x$; $F_y = F\cos\theta_y$ $F_z = F\cos\theta_z$
Magnitude	$F = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$
Mag and Dir. **F** Line of Action of **F** from Pt A to Pt B	$\underline{\mathbf{F}} = F\underline{\mathbf{n}_F}$ $\underline{\mathbf{n}}_F = \frac{\overrightarrow{AB}}{AB} = \frac{(x_B-x_A)\hat{\imath} + (y_B-y_A)\hat{\jmath} + (z_B-z_A)\hat{k}}{\sqrt{(x_B-x_A)^2 + (y_B-y_A)^2 + (z_B-z_A)^2}}$ $= (\cos\theta_x)\hat{\imath} + (\cos\theta_y)\hat{\jmath} + (\cos\theta_z)\hat{k}$
Angle of force above plane Component of force in plane (here x-y plane)	ϕ = angle above x-y plane θ = angle in x-y plane from x-axis Component in plane: $F_{xy} = F\cos\phi$ Normal plane: $F_z = F\sin\phi$ $F_x = F_{xy}\cos\theta = F\cos\phi\ \cos\theta$ $F_y = F_{xy}\sin\theta = F\cos\phi\ \sin\theta$

Dot Product α = angle between vectors

$$\underline{\mathbf{P}}\cdot\underline{\mathbf{Q}} = PQ\cos\alpha = P_xQ_x + P_yQ_y + P_zQ_z$$
$$\cos\alpha = \frac{\underline{\mathbf{P}}\cdot\underline{\mathbf{Q}}}{PQ}$$

Projection of **F** onto direction of unit vector **n**:

$$\underline{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} = F[(\cos\theta_x)\hat{\imath} + (\cos\theta_y)\hat{\jmath} + (\cos\theta_z)\hat{k}]$$
$$\underline{\mathbf{n}} = n_x\hat{\imath} + n_y\hat{\jmath} + n_z\hat{k}$$
$$\underline{\mathbf{n}} = [(\cos\theta_{nx})\hat{\imath} + (\cos\theta_{ny})\hat{\jmath} + (\cos\theta_{nz})\hat{k}]$$
$$F_n = \underline{\mathbf{F}}\cdot\underline{\mathbf{n}} = F_xn_x + F_yn_y + F_zn_z$$
$$F_n = \underline{\mathbf{F}}\cdot\underline{\mathbf{n}} = F[(\cos\theta_x)(\cos\theta_{nx}) + (\cos\theta_y)(\cos\theta_{ny}) + (\cos\theta_z)(\cos\theta_{nz})]$$
$$\cos\alpha = \frac{\underline{\mathbf{F}}\cdot\underline{\mathbf{n}}}{F}$$

Moment and Couple

$$\underline{\mathbf{r}} = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k} \ ; \ \underline{\mathbf{F}} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$
$$\underline{\mathbf{M_O}} = \underline{\mathbf{r}} \times \underline{\mathbf{F}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$
$$\underline{\mathbf{M_O}} = (r_yF_z - r_zF_y)\hat{\imath} + (r_zF_x - r_xF_z)\hat{\jmath} + (r_xF_y - r_yF_x)\hat{k}$$
$$\underline{\mathbf{M_O}} = M_{Ox}\hat{\imath} + M_{Oy}\hat{\jmath} + M_{Oz}\hat{k}$$
$$M_O = \sqrt{{M_{Ox}}^2 + {M_{Oy}}^2 + {M_{Oz}}^2}$$

In any direction, λ, projection of $\underline{\mathbf{M_o}}$ onto λ-axis, where **n** is unit vector in λ-direction:

$$M_{O\lambda} = \underline{\mathbf{M_O}}\cdot\underline{\mathbf{n}} = (\underline{\mathbf{r}} \times \underline{\mathbf{F}})\cdot\underline{\mathbf{n}}$$
$$\underline{\mathbf{M_{O\lambda}}} = M_{O\lambda}\underline{\mathbf{n}} = (\underline{\mathbf{r}} \times \underline{\mathbf{F}}\cdot\underline{\mathbf{n}})\underline{\mathbf{n}}$$

Equivalent Force-Couple Systems ... $\underline{\mathbf{R}} - \underline{\mathbf{M_O}}$)

$$\underline{\mathbf{R}} = \sum\underline{\mathbf{F}} \ ; \ \underline{\mathbf{M_O}} = \sum\underline{\mathbf{M_O}} = \sum(\underline{\mathbf{r}} \times \underline{\mathbf{F}}) + \sum\underline{\mathbf{M_{Couples}}}$$
$$\underline{\mathbf{R}} = (\sum F_x)\hat{\imath} + (\sum F_y)\hat{\jmath} + (\sum F_z)\hat{k} = R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$$
$$R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$$
$$\sum\underline{\mathbf{M_O}} = \left[\sum(\underline{\mathbf{r}} \times \underline{\mathbf{F}})_x\right]\hat{\imath} + \left[\sum(\underline{\mathbf{r}} \times \underline{\mathbf{F}})_y\right]\hat{\jmath} + \left[\sum(\underline{\mathbf{r}} \times \underline{\mathbf{F}})_z\right]\hat{k}$$
$$= M_{Ox}\hat{\imath} + M_{Oy}\hat{\jmath} + M_{Oz}\hat{k}$$
$$M_O = \sqrt{{M_{Ox}}^2 + {M_{Oy}}^2 + {M_{Oz}}^2}$$

Chapt 3 Equilibrium

FBDs!

2D Supports; Possible Reactions (x-y plane)

Pin	A_x, A_y
Roller/Rocker; Slider	Normal force
Fixed/Built-In (wall)	A_x, A_y, M_A
Smooth Surface	Normal force
Rough Surface	Normal force, Friction force
Cable; Axial-only member	Force in direction of cable

2D Equilibrium

$$\underline{\mathbf{R}} = \sum \underline{\mathbf{F}} = 0 \quad \sum \underline{\mathbf{M}_O} = \mathbf{0}$$

I	II	III
$\sum F_x = 0$	$\sum F_n = 0$	$\sum \mathrm{M}_A = 0$
$\sum F_y = 0$	$\sum \mathrm{M}_A = 0$	$\sum \mathrm{M}_B = 0$
$\sum \mathrm{M}_A = 0$	$\sum \mathrm{M}_B = 0$	$\sum \mathrm{M}_C = 0$

3D Supports; Possible Reactions

Ball-Socket	A_x, A_y, A_z
Roller with lateral constraint	Normal force, Lateral force
Fixed/Built-In (wall)	A_x, A_y, A_z, M_{Ax}, M_{Ay}, M_{Az}
Thrust Bearing, shaft along x-axis	A_x, A_y, A_z, M_{Ay}, M_{Az}
Simple Bearing, shaft along x-axis	A_y, A_z, M_{Ay}, M_{Az}
Two Bearings, A, B, shaft along x-axis, A is thrust, B is simple	A_x, A_y, A_z, B_y, B_z
Smooth Surface	Normal force
Rough Surface	Normal force Friction force

3D Equilibrium

$\underline{\mathbf{R}} = \sum \underline{\mathbf{F}} = 0$	$\sum \underline{\mathbf{M}_A} = \mathbf{0}$
$\sum F_x = 0$	$\sum \mathrm{M}_{Ax} = 0$
$\sum F_y = 0$	$\sum \mathrm{M}_{Ay} = 0$
$\sum F_z = 0$	$\sum \mathrm{M}_{Az} = 0$

Chapt 4 Structures FBDs!

Truss: An assembly of two-force members pinned together. All loads applied at joints.

Method of Joints (2D)

0. Solve reactions of entire truss (not always)
1. Isolate a joint with only two unknown forces acting on it.
2. Apply equilibrium to joint
   $$\sum F_x = 0; \quad \sum F_y = 0$$
3. Repeat Steps 1 to 2 as needed.

Method of Sections (2D)

0. Solve reactions of entire truss (not always)
1. Consider a section cut through truss members, exposing no more than three unknowns. Do not cut though joints.
2. Apply equilibrium to part of the truss to one side of the section. Select one of 3 sets of 2D equilibrium equations. Use a 4th equation to check.
3. Repeat Steps 1 to 2 as needed.

Frame/Machine: An assembly of members pinned together. At least one member is not a two-force member.

For Rigid-Collapsible Frames/Machines:

1. Break system up into individual FBDs. Apply Newton's 3rd Law.
2. Apply equilibrium on each member, solving pin forces by possibly considering more than one member at a time.
3. Repeat Steps 1 to 2 as needed.

Hint: look for the two-force member.

Hint2: the force inside a multi-force member is not generally in the direction of the member.

Chapt 5 Distributed Forces

Centroids of Lines, Areas, Volumes

$L = \int dL$	$\bar{x} = \frac{\int x_c dL}{L}$; $\bar{y} = \frac{\int y_c dL}{L}$; $\bar{z} = \frac{\int z_c dL}{L}$
$A = \int dA$	$\bar{x} = \frac{\int x_c dA}{A}$; $\bar{y} = \frac{\int y_c dA}{A}$; $\bar{z} = \frac{\int z_c dA}{A}$
$V = \int dV$	$\bar{x} = \frac{\int x_c dV}{V}$; $\bar{y} = \frac{\int y_c dV}{V}$; $\bar{z} = \frac{\int z_c dV}{V}$

Composite Areas and Bodies

$A = \sum A_i$	$\bar{x} = \frac{\sum \bar{x}_i A_i}{A}$; $\bar{y} = \frac{\sum \bar{y}_i A_i}{A}$; $\bar{z} = \frac{\sum \bar{z}_i A_i}{A}$
$V = \sum V_i$	$\bar{x} = \frac{\sum \bar{x}_i V_i}{V}$; $\bar{y} = \frac{\sum \bar{y}_i V_i}{V}$; $\bar{z} = \frac{\sum \bar{z}_i V_i}{V}$

Beams

Members that resist bending due to applied loads. Generally long bars with forces applied normal to the member's structural axis.

Distributed Loading, $w(x)$

Total Load	$F_{eq} = \int w(x)dx$
Location	$\bar{x} = \frac{\int x[w(x)dx]}{F_{eq}}$

Shear Force and Bending Moment Distributions

Consider lengths of beam cut at various x.

Number of cuts: 1 + number of times the load on the beam changes

0. FBD of entire beam, solve reactions
1. Cut beam at length x.
2. Apply equilibrium to segment of length x:
   $\sum F_y = 0$; $\sum M = 0$ … solve $V(x)$ and $M(x)$
3. Solve shear force $V(x)$ and bending moment $M(x)$ over the entire length of the beam.

$V = \frac{dM}{dx}$	$\Delta M = M_2 - M_1$ = area under the shear-force diagram from x_1 to x_2, not including point couples
$w = -\frac{dV}{dx}$	$\Delta V = V_2 - V_1$ = negative of the area under the shear-force diagram from x_1 to x_2, not including point forces

Shear Force and Bending Moment Diagrams

1. Plot shear force and bending moment diagrams (plot to-scale).
2. Label values at ends of beam, where $V(x)$ and $M(x)$ change form, and where $V(x)$ and $M(x)$ are maximum and minimum.

Cables

Parabolic Cable: $w(x) = w$ = constant,
T_0 = minimum tension (at bottom of cable)

$y = \frac{wx^2}{2T_0}$	$T_0 = \frac{wl_A^2}{2h_A}$; $T_0 = \frac{wl_B^2}{2h_B}$

$$T = \sqrt{T_0^2 + w^2x^2} = w\sqrt{x^2 + (l_A^2/2h_A)^2}$$
$$T_{max} = wl_A\sqrt{1 + (l_A/2h_A)^2}$$

For $h_A/l_A < ½$, length of cable from bottom of sag to $(x,y)=(l_A, h_A)$

$$s_A = l_A\left[1 + \frac{2}{3}\left(\frac{h_A}{l_A}\right)^2 - \frac{2}{5}\left(\frac{h_A}{l_A}\right)^4 + \cdots\right]$$

For symmetric parabolic cable, sag h, span L.

Maximum Tension	$T_{max} = \frac{wL}{2}\sqrt{1 + (L/4h)^2}$
Total Cable Length	$S = L\left[1 + \frac{8}{3}\left(\frac{h}{L}\right)^2 - \frac{32}{5}\left(\frac{h}{L}\right)^4 + \cdots\right]$

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