Electrical circuit problems are generally solved using systems of linear equations.

Consider the electric circuit shown in *Figure 1*. It consists of one *voltage source* (12 V) and three *resistors* ($R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$ and $R_3 = 6\ \Omega$). The resistor values have units of *ohms* (Ω). Each electrical *element* (voltage source or resistor) is a *branch* and an electrical current flows through each element: i_1, i_2 and i_3. *Current* is the flow of electric charge per time; the units of current are *amperes* or *amps* (A). *Nodes* connect branches; there are three nodes in this circuit, which have been labelled A, B and C.
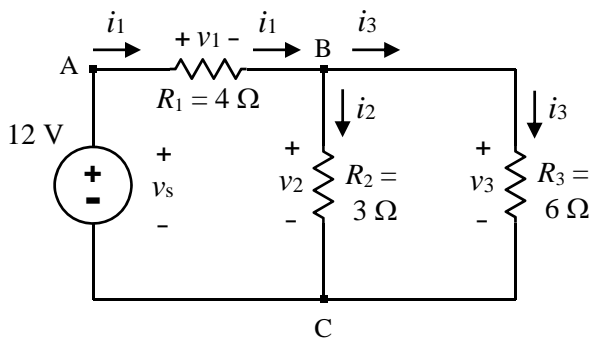
Figure 1

A *voltage rise* is the energy (work) required per unit charge to move the charge through an element (e.g., a voltage source, such as a battery, usually provides energy to the circuit). In other words, a *voltage rise* is the increase in electric potential energy of a unit charge. Conversely, a *voltage drop* is the loss of energy per unit charge as it passes through an element (e.g., in a resistor, electric energy is removed from the circuit and lost as heat).

When the voltage changes are labeled across an element (e.g., "+ v_1 –" in *Figure 1*), the positive (+) terminal (Node A) is assumed to be at a higher voltage that the negative (–) terminal (Node B). The voltage is assumed to drop from left to right across R_1. In your calculations, if voltage drop v_1 turns out positive, the voltage drop is in the direction assumed (from A to B); if v_1 turns out negative, it is opposite assumed (from B to A).

In a circuit problem, usually the voltage sources and resistances are given, and we need to find the current i in each branch and the voltage drop v across each resistor. Our has six unknowns: i_1, i_2 and i_3, and v_1, v_2 and v_3. While we could develop a system of six equations for this problem, we will reduce it to three equations before letting MATLAB do the work.

There are three fundamental circuit laws: *Ohm's Law*, and *Kirchoff's Voltage* and *Current Laws*.

1. Ohm's Law

Ohm's Law states that the *voltage drop* across a resistor is proportional to the *current* through it:
$$v = iR \qquad (1)$$
For example, the voltage drop across Resistor 1, v_1, (from Node A to Node B) is:
$$v_1 = i_1 R_1 = 4i_1$$
Engineering convention is that the voltage drop across a resistor is in the direction of current flow. To model this mathematically, the voltage drop (from + to – terminal) is $v = +iR$ if current i is drawn from + terminal to – terminal. In this example, we will always draw i from + terminal to – terminal, so v is always $+iR$ (Sometimes, it is not convenient to draw i into the positive terminal of the voltage drop; if i enters the – terminal, $v = -iR$).

tive (from (opposite assumed; e.g., Node A would be at a lower voltage than Node B).

By using Ohm's Law, the number of unknowns in this problem can be reduced to three (the currents).

2. Kirchoff's Voltage Law

Kirchoff's Voltage Law (KVL) states that the algebraic sum of the voltage drops around a *loop* (a closed path) is zero:
$$\sum v_i = 0 \qquad (2)$$
As you move (trace a path) around a loop, the sign of the voltage drop is taken as:

- *positive* if you enter the assigned *positive terminal*;
- *negative* if you enter the assigned *negative terminal*.

The circuit in *Figure 1* has three loops: 12V-R_1-R_2, R_2-R_3 and 12V-R_1-R_3. Note that the left loop and right loop can be combined to form the outside loop (12V-R_1-R_3).

Around the left loop:
$$\sum v_i = -v_s + v_1 + v_2 = -(12) + (4i_1) + (3i_2) = 0$$

3. Kirchoff's Current Law

Kirchoff's Current Law (KCL) states that the algebraic sum of the currents at any node is zero:
$$\sum i_i = 0 \qquad (3a)$$
Assigning incoming currents as positive, and outgoing currents as negative, at Node B we have:
$$i_1 - i_2 - i_3 = 0$$
Alternatively, we could express KCL as:
$$\sum i_{in} = \sum i_{out} \qquad (3b)$$
e.g.:
$$i_1 = i_2 + i_3$$

For the circuit in *Figure 1*, we need to develop three equations to solve for the three unknowns: i_1, i_2 and i_3.

The first two equations come from Kirchoff's Voltage Law. Apply KVL by taking the sum of the voltage drops as you go around the Left and Right Loops, respectively, recalling the voltage drop is written positive when entering the + terminal, and negative when entering the – terminal. A negative voltage drop is a voltage rise.

For the voltage drops across resistors, we will then rewrite them using Ohm's Law (e.g.: $v=Ri$).

Left Loop (clockwise, from Node C to A to B to C):
$$\sum v_i = -v_s + v_1 + v_2 = -(12) + (4i_1) + (3i_2) = 0 \quad (4)$$

There is a negative in front of v_s since as we go from Node C to A, the voltage rises by v_s; it is a negative voltage drop.

Right Loop (clockwise, from Node C to B to C):
$$\sum v_i = -v_2 + v_3 = -3i_2 + 6i_3 = 0 \quad (5)$$

There is a negative in front of the v_2 in the Right Loop equation since as we move clockwise around the loop, we are going entering – terminal of v_2.

Consider going around the Outer Loop (12V- R_1- R_3):
$$\sum v_i = -v_s + v_2 + v_3 = -12 + 4i_2 + 6i_3 = 0$$

Physically, we are combining the Left and Right Loops. Mathematically, we are adding the loop equations, Equations (4) and (5), which do not create a new independent equation.

We need one more equation, and that is found by applying KCL at either Node B or C (where 3 branches meet):
$$i_1 = i_2 + i_3 \quad (6)$$

Writing out Equations (4)–(6) as a system of equations:
$$4i_1 + 3i_2 = 12$$
$$-3i_2 + 6i_3 = 0 \quad (7)$$
$$i_1 - i_2 - i_3 = 0$$

and then in matrix form:
$$\begin{bmatrix} 4 & 3 & 0 \\ 0 & -3 & 6 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

In matrix notation, the system of equations is, in general:
$$\mathbf{Ax = b} \quad (9)$$

where $\mathbf{A}$ is the coefficient matrix, $\mathbf{x}$ is the current vector, and $\mathbf{b}$ is the constant vector. The currents are solved by pre-multiplying both sides by the inverse of $\mathbf{A}$:
$$\mathbf{x = A^{-1}b} \quad (10)$$

Note: if we did not manually apply Ohm's Law to reduce all of the equations to currents, we would generate a 6x6 system of equations for the above problem.

EXERCISES

**Engr 170 students: use *branch currents*, do not us *mesh currents*.**

Exercise 1:

Using MATLAB, solve for the currents i_1, i_2 and i_3 in the circuit in *Figure 1*. The units of the currents are in amps (A).

Exercise 2:

a. Generate a system of linear equations for the circuit shown in *Figure 2*. Note that there are 6 different currents.

   Hint: Label the voltage drop across each resistor in the direction of its current (as in *Fig.1*). Do KVL around three loops, and KCL at Nodes A, B and C. Also recall the sign of the voltage drop when entering the positive or negative terminal.

b. Using MATLAB, solve for the currents i_1 through i_6. The units will be in amps (A).
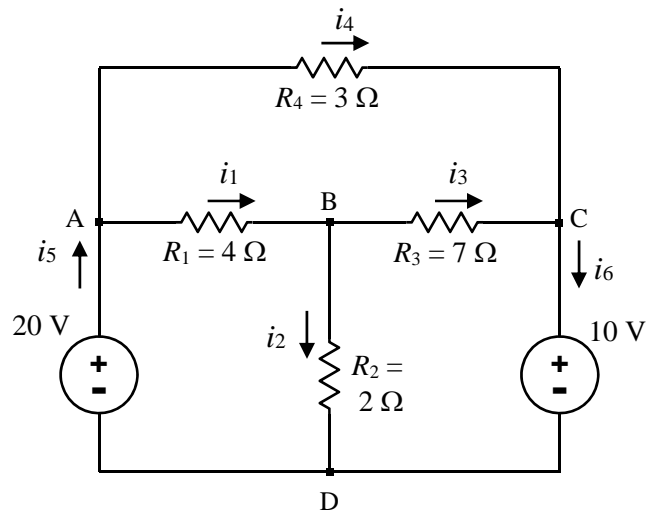
Figure 2

SOLUTIONS (write the currents)

Exercise 1

	Current (A)
i_1	
i_2	
i_3	

Exercise 2 (i_6 is given to help check your work)

	Current (A)
i_1	
i_2	
i_3	
i_4	
i_5	
i_6	2.9333